

Math 180, Quiz 8: Thursday, March 21, 2013

Take fifteen minutes to complete this quiz. Please show all your work, and write your name on the front and back of the paper before turning it in. Make sure to show all relevant work.

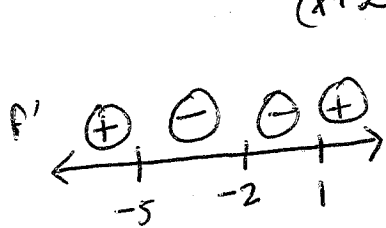
1. (9 pts) Sketch a graph of the function $f(x) = \frac{x^2 + 5}{x + 2}$. In particular:

- Make sure your sketch is increasing and decreasing in the right places and has the appropriate concavity and end behavior.
- Indicate any critical and/or inflection points on the graph, including their coordinates.
- Indicate any asymptotes with dashed lines.
- Write down all calculations needed to establish the features above.

(They probably won't give you this checklist on the exam, but I thought I'd be nice and add it here.)

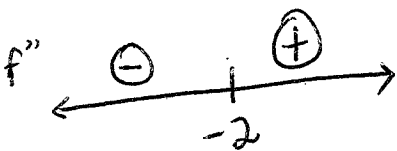
$$f'(x) = \frac{(x+2)(2x) - (x^2+5)(1)}{(x+2)^2} = \frac{2x^2+4x-x^2-5}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2} = \frac{(x-1)(x+5)}{(x+2)^2}$$

$$f''(x) = \frac{(x+2)^2(2x+4) - (x^2+4x-5)2(x+2)}{(x+2)^4} = \frac{(2x^2+8x+8) - (2x^2+8x-10)}{(x+2)^3} = \frac{18}{(x+2)^3}$$



$$f(1) = \frac{1+5}{1+2} = \frac{6}{3} = 2 \text{ (local min)}$$

$$f(-5) = \frac{25+5}{-5+2} = \frac{30}{-3} = -10 \text{ (local max)}$$



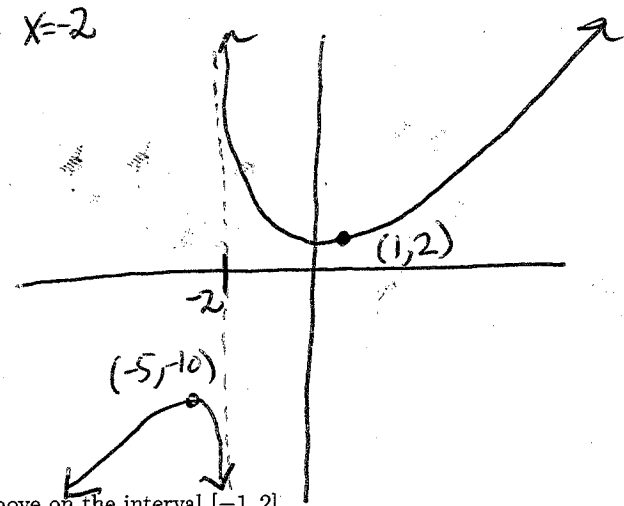
$$\lim_{x \rightarrow -2^+} \frac{x^2+5}{x+2} = +\infty$$

$$\lim_{x \rightarrow -2^-} \frac{x^2+5}{x+2} = -\infty$$

VA $x = -2$

$$\lim_{x \rightarrow \infty} \frac{x^2+5}{x+2} = \lim_{x \rightarrow \infty} \frac{x(1+\frac{5}{x})}{x(1+\frac{2}{x})} = \lim_{x \rightarrow \infty} \frac{x+5/x}{1+2/x} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2+5}{x+2} = \lim_{x \rightarrow -\infty} \frac{x+5/x}{1+2/x} = -\infty$$



2. (1 pt) Find the absolute extrema of the function $f(x)$ above on the interval $[-1, 2]$.

$$f(-1) = \frac{1+5}{-1+2} = \frac{6}{1} = 6 \text{ ← absolute max}$$

$$\text{(from above)} \quad f(1) = 2 \text{ ← absolute min}$$

$$f(2) = \frac{4+5}{2+2} = \frac{9}{4}$$